MATHEMATICAL MODELING OF THE INDUCTIONAL TEMPERING OF STEEL SAMPLES

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Complete mathematical modeling of processes of inductional tempering in various inductors is performed on a computer. Using regularizing algorithms, the problem of heat control and the problem of purposive control are solved.

1. As is known, the method of mathematical modeling of physical and, in particular, technological processes is an effective means of studying the laws controlling them, or predicting the results of the process as a function of the controlling parameters. This method involves the development of a mathematical model of the process reflecting its significant features and the performance of a numerical experiment on a computer, reproducing the whole process.

Often, all the physical factors controlling a process are not known in advance. Then inverse and usually incorrect problems must be solved in order to develop a mathematical model of the process. In this case, an intrinsic part of the construction of the model is the development of stable algorithms, and the basis for this is the theory of regularization [1], the application of which to thermophysical problems has been developed in a series of works (e.g. [2]).

In the present work, for the example of inductional tempering, the features of problems of modeling technological processes are elucidated.

2. The process of inductional tempering of steel samples in an inductor carrying a variable current $I = I(t)e^{-i\omega t}$ includes two phases: Heating of layers of the sample, beginning with the surface, to the temperature of complete austenitic conversion; and rapid cooling, performed by immersing the sample surface in a fast liquid flow.

The main problem in developing a model of the first phase is to determine the law of variation of the current amplitude in the inductor I(t) so that the temperature conditions of the sample surface meet a definite requirement (i.e., $T|_S=\hat{T}$), ensuring the necessary heating. This is an inverse problem of control type. The a priori requirements on the conditions may be sufficiently rigorous, but "competence" [1] of the apparatus (i.e., the existence of at least one appropriate control) is possible as a result of tolerance (i.e., $\|T|_S-\hat{T}\|<\delta$) in the deviation of the surface conditions from those required. In turn, the uniqueness of the control is unimportant. It is important, however, that I(t) satisfy definite conditions of practical realizability $I(t)\in K$.

As is known [1], "regularizing" algorithms are those which allow as accurate an approximation to some unique solution of the problem as desired to be obtained, with a sufficiently small error of its input data. In control problems with "accurate" input data, the existence of any solution at all often cannot be guaranteed. At the same time, the problem is not stable even in this case — no explicit algorithm leads to a realizable (and even physically meaningful) "solution." Algorithms leading to a solution with the necessary properties — e.g., $I(t)\in K$) — and stability with respect to change in the input data will be called "conditionally regularizing" for the control problem.

The development of a model for the second phase of tempering requires refinement of the heat-transfer law at the sample surface (because of the complexity of the physical processes associated with the given means of cooling). This is an inverse problem of the type of the interpretation of physical observation data (in the given case, above the surface temperature

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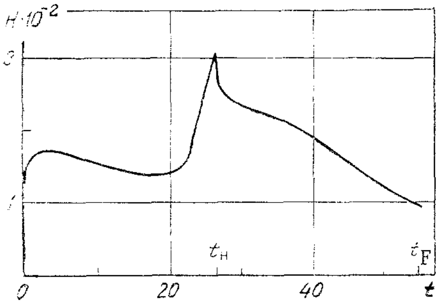


Fig. 1

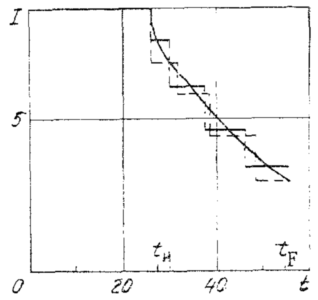


Fig. 2

Fig. 1. Control of the heating (magnetic field) for a "solenoidal" inductor. H, A/mm; t, sec.

Fig. 2. Control of the heating for a "band" inductor, I, kA.

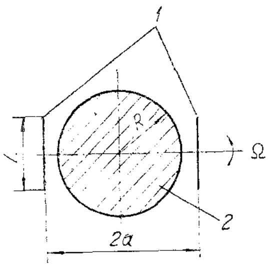


Fig. 3

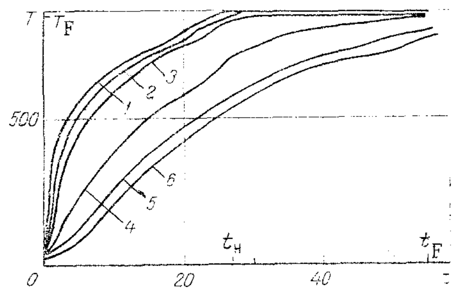


Fig. 4

Fig. 3. Inductor of "band" type: 1) inductor busbars; 2) sample (R = 24 mm, L = 28 mm, a = 27 mm).

Fig. 4. The temperature field for heating in a "band" inductor. The temperature at the surface: maximum (1), mean (2), minimum (3); (mean) temperature at r = 16.4 mm (4) and 8.8 mm (5); temperature at the center of the sample (6).

of the sample) solved in [3], and we will not dwell on the corresponding results here. For this problem, the uniqueness of the solution (with accurate input data) is significant, since identification of the model with the real object would otherwise be impossible.

If a model of the two phases of induction is constructed, then one further inverse problem may be solved by mathematical experiment — the determination of certain numerical parameters of the operating conditions in which the greatest tempering effect is obtained. This problem also permits a mathematical formulation as a problem of optimizing some object functional. Since the problem of the character of the controlling function is solved in the course of the experiment, regularizing or conditionally regularizing algorithms are a necessary element of the optimization complex. If such a complex is constructed, then programmatic control of the technological process may be realized.

3. For the problem of controlling heating under tempering, a two-dimensional model is considered (a sample of "infinite" length), and correspondingly it is assumed that, regardless of its construction, the inductor creates a field with one component (H or E) parallel to the sample axis. The temperature field in the sample is described by the equation

$$\operatorname{div}(k(T)\operatorname{grad}T) + 0.12\,\sigma(T)\,|j|^2 = c(T)\,\rho(T)\,\frac{\partial T}{\partial t},$$

$$0 < r < R,\quad 0 \leqslant \varphi < 2\pi,\quad 0 < t \leqslant t_F, \tag{1}$$

where k(T), c(T), ρ(T), σ(T) are functions of the temperature T; j is the mean value over the period of the unique component of the current density in the sample, and is a function of I(t): $j = \Phi(r, \varphi, I(t))$; its value is determined by a stream of Maxwell equations with additional conditions corresponding to the type of inductor, so that the temperature field — at each I(t) — is ultimately determined by a system of nonlinear partial differential equations. This system is closed by means of additional conditions on the temperature field

$$T|_{t=0} = T_0, \ \lim_{r \to 0} rk(T) \frac{\partial T}{\partial r} = 0, \ -k(T) \frac{\partial T}{\partial r}\bigg|_{r=R} = h_0 (T - T_0)|_{r=R}. \tag{2}$$

Then at each I(t) the temperature field may be calculated on a computer using difference schemes [4], and thus the operator Ψ is algorithmically specified

$$\Psi(t, I(t)) = \frac{1}{2\pi} \int_0^{2\pi} T(R, \varphi, t)\, d\varphi, \tag{3}$$

which determines the behavior of the mean temperature at the sample surface. Below, "local" operators will also be considered: $\Psi_S(t, I(t))$, $t \in \Delta_s \equiv [t_{s-1}, t_s]$, defined in contrast to Ψ, by the condition $T|_{t=t_{s-1}} = T(t_{s-1}, r, \varphi)$ for any particular segment $\Delta_s \in [0, t_F]$.

The inverse problem of finding I(t) with specific requirements regarding the temperature conditions at the boundary may be "regularized" on the basis of various a priori constraints on the desired function, corresponding to the control possibilities. The basis requirement on the surface temperature conditions is the provision of two stages of heating: fast (e.g., linear) heating to $T_F \sim 900°C$, exceeding the temperature of complete austenitic conversion ($Ac_3 \sim 790°C$); and isothermal holding of the surface at $T = T_F$. Of course there is a tolerance δ in the temperature deviations of the surface from the required value ($\delta \sim 10°C$), which will be taken into account in the mean-square approximation.

Problem 1. To find the piecewise-monotonic control I(t) satisfying the condition

$$\rho^2(\Psi, \hat{\Psi}) \equiv \int_0^{t_F} [\Psi(t, I(t)) - \hat{\Psi}(t)]^2\, dt \leqslant \delta^2, \tag{4}$$

where $\hat{\Psi}(t)$ is the already-specified surface temperature. The conditionally regularizing algorithm for solving this problem is given by the formulas [5]

$$I_k : \min (I - I_{k-1}^0)^2, \quad I \in j^{(k)} \equiv \left\{ I : \int_{t_{k-1}}^{t_k} [\Psi_k(t, I) - \hat{\Psi}(t)]^2\, dt \leqslant \delta_k^2 \right\},$$
$$k = 1, 2, \ldots, n, \quad \sum_{k=1}^{n} \delta_k^2 = \delta^2, \tag{5}$$

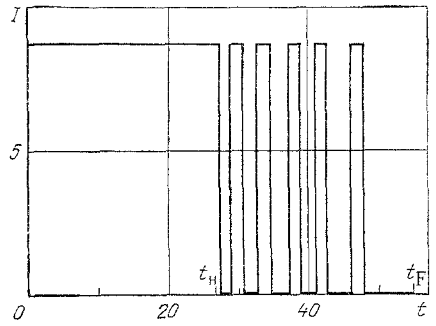
where I_{k-1}^0 is the solution of the problem in Eq. (5) in the preceding segment (in the calculations, $n \sim 300$). An example of the solution of the problem using this algorithm is shown in Fig. 1 for a "solenoidal" inductor creating a longitudinal magnetic field ($j = j_\varphi$, $\Phi = \partial H_z / \partial r$).

Problem 2. a) For the stage of rapid heating, to find the constant value of the current I such that the mean surface temperature in Eq. (3) takes the specified value T_F at time $t = t_H$; b) for the stage of isothermal holding, $t_H \leqslant t \leqslant t_F$, to find the control I(t). In Fig. 2, the control found [6] for another type of inductor (Fig. 3) is shown. In this case the sample rotates about its axis in the transverse magnetic field of the inductor, depending on the two spatial variables (r, φ). Now $j = j_Z = \sigma E_Z$, and the function Φ is determined by the solution of the problem of inductor-field diffraction in the cylinder.

If $f(I) \equiv \Psi(t_H, I)$ is a functional of I, then problem 2a reduces to the equation: $f(I) = T_F$, $I \in E^1$. It is conditionally correct [7, 1] in view of the monotonicity of f(I), and is solved using the chordal method of [8]. Problem 2b is analogous to Eq. (4), and is solved by means of Eq. (5). The behavior of the temperature at the surface and axis of the sample is shown in Fig. 4. Such diagrams may serve, in particular, for the selection of heating times that ensure sample heating which is relatively uniform with respect to φ.

Problem 3. On the basis of the solution of problem 2a, to find a "multistep" control with a minimum number of elements m for the stage of isothermal holding.

The mathematical formulation of this problem, e.g., for a "band" inductor (Fig. 3), is as follows: to find t_k, dividing the segment $[t_H, t_F]$ and the values of I_k at $t \in \Delta_k$ from the condition



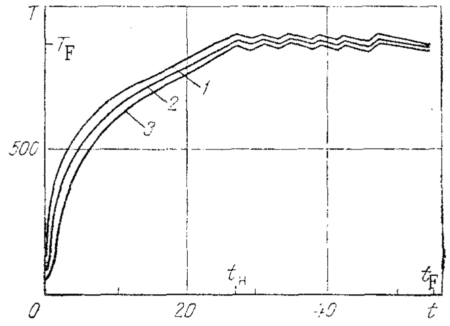


Fig. 5 Fig. 6

Fig. 5. "Pulsed" control of the heating in a "band" inductor.

Fig. 6. Surface temperature with "pulsed" control: 1) maximum; 2) mean; 3) minimum.

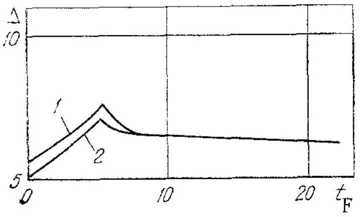


Fig. 7. Tempered-layer thickness for a "band" inductor: 1) maximum; 2) minimum. Δ, mm.

$$M = \min n(I(t)) : I(t) \in V_\delta \equiv \{\{I_k,\ t \in \Delta_k,\ k = \overline{1, \ldots, m}\},$$
$$0 \leqslant I_k \leqslant M, \quad \rho^2(\Psi, \hat{\Psi}) \leqslant \delta^2\}. \tag{6}$$

Suppose that the "continuous" control satisfying the condition in Eq. (4) has already been found using Eq. (5). Then the problem in Eq. (6) reduces to the approximation of the specific continuous function by some multistep function with a minimum number of elements but such that the condition $\rho^2(\Psi, \hat{\Psi}) \leqslant \delta^2$ is satisfied. The search algorithm is described in [9], and we will not consider it here. The control found is also shown in Fig. 2 by a continuous curve, together with other possible controls (the steps — dashed curves) obtained by direct sequential search for pairs (I_j, t_j), j = 1, 2, ..., s − 1 under the condition $\max_t |\Psi(t, I) - T_F| \leqslant \delta$ $(t \in [t_{s-1}, t_s])$.

Problem 4. To find a "pulsed" control for the stage of isothermal holding. Let I_0 be the current found for the stage of rapid heating and K_N be a set of piecewise-constant functions that correspond to switching on the current I_0 at time t_{2m} and switching off at t_{2m+1}. The division $\{\Delta_k\}$ with the minimum possible number of elements N will be sought, such that the minimum and maximum surface temperatures do not fall outside the range of δ_n:

$$\min N(I(t)) : I(t) \in \overline{V_{\delta_n}} \equiv$$
$$\equiv \{I(t) \in K_N,\ \max_{t_H \leqslant t \leqslant t_F}\ \max_{0 \leqslant \varphi \leqslant 2\pi} |T(R, \varphi, t) - T_F| \leqslant \delta_n\}.$$

The algorithm for solving this problem is similar to the second variant of the search for a solution of the problem in Eq. (3), the difference being that what is selected is, in turn: a) pairs (I_0, t_s) and b) pairs $(0, t_{s+1})$, such that the values of the boundary temperature do not fall outside the tolerance δ.

The selected pulsed conditions ($\delta_n \simeq 25\,°C$) are shown in Fig. 5, and the corresponding temperature field is shown in Fig. 6. The results described above evidently allow the required control conditions for sample heating to be selected. The problem of determining the heat-transfer law at the surface with rapid cooling was solved in [3] using the algorithm in Eq. (5).

4. The final step is to consider the results of complete modeling of the tempering process obtained with a fixed heat-transfer law H_0 = 100,000 kcal/$m^2 \cdot h \cdot {}^\circ C$, characterizing the thickness of the tempered [3] surface layer Δ. The value of Δ is determined by introducing thermokinetic diagrams into the computer [10] and comparing them with the temperature curves of cooling (this comparison is completely automated [3]).

Nomograms of the dependence of Δ on the time of isothermal holding for a band inductor are shown in Fig. 7. In view of the nonuniformity of heating with respect to φ, two curves corresponding to the maximum and minimum layer thickness with respect to φ are shown. It may be seen that: a) After a certain holding time, the depth of tempering reaches a value that is a maximum and is uniform in φ; b) increases in holding time right up to through-heating of the sample does not lead to any benefit in terms of the tempered-layer thickness. This result of the competition of two fluxes — cooling from the surface and warming (from within the rod in deep heating) — cannot be predicted without mathematical modeling, at least qualitatively. The results obtained have been introduced at the AVTO-ZILa factory, where the conditions of tempering samples of the given type have been corrected. At the department of mathematics in the physics faculty of Moscow State University, packets of programs solving the above-considered complex of programs on a computer have been developed.

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